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# Advanced Free-rider Attacks in Federated Learning

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## Abstract

Federated learning is a new machine learning technology that multiple clients collaboratively to train a global model without sharing their local data. Due to the fact that clients have the direct control over their local models and training data, federated learning is inherently vulnerable to free-rider attacks that the malicious client forges local model parameters to get reward without contributing sufficient local data and computation resources. Recently, many different free-rider attacks have been proposed. However, existing attacks haven't a good stealth property. The convergence property represents the convergent speed and final global model accuracy. The stealth property indicates the attacker's ability to hide its local update. In this work, we first utilize the Ornstein-Uhlenbeck (OU) process to formalize the evolution of local and global training processes, and analyze the geometrical relationship of all clients' local model updates. Then, we propose a scaled delta attack and an advanced free-rider attack. We also prove that advanced free-rider attack can not only ensure the convergence of the aggregated model, but also hold the stealth property. Experiment results demonstrate that our advanced free-rider attack is feasible and can escape from state-of-the-art defense mechanisms. Our results show that even a highly constrained adversary can carry out the advanced free-rider attack while simultaneously maintaining stealth under the defense strategies, which highlights the vulnerability of the federated learning setting and the need to develop effective defense strategies.

## 1 Introduction

Federated learning (FL) is a popular implementation of distributed stochastic optimization for large-scale deep neural network training [1, 2, 3]. It is a multi-round strategy where multiple clients work together to train a model under the orchestration of a central server, while preserving the confidentiality of the local training data. To ensure clients privacy, federated learning is designed to have no visibility into clients local data and training processes. FL systems can preserve data privacy and reduce the costs resulting from traditional centralized machine learning frameworks. Nowadays, more and more federated learning frameworks have been developed and deployed, such as TensorFlow Federated Framework and Webank's FATE.

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In recent years, the security of FL has received significant interest from both research communities and the industry. There has been intensive work focusing on attacks and defenses for various attack vectors, such as evasion attack, data poisoning attack and model poisoning attacks [3]. In evasion attacks, the adversary alters the data used at inference-time. In data poisoning attacks, an adversary mainly injects malicious data into the training dataset before the learning process starts, while the learning process is assumed to maintain integrity [4, 5, 6]. In model poisoning attacks [7, 8], an adversary exploits the fact that a client can directly manipulate local model update sent back to the server. Free-rider attacks [8, 9] a new special attack method, where free-riders construct local updates through the sharing of opportune counterfeited parameters. The free-riders may submit fake parameters due to several reasons: 1) the adversary does not have any local data for training; 2) the adversary wants to save local storage spaces or computation resources [10].

In this paper, we propose advanced free-rider attack where the adversary can carefully construct local model update without being detected. We first formulate the local and global training processes of FL with OU process. Then, we propose our scaled delta attack that holds the convergence property for the aggregated global model but lacks a good stealth property. After that, we analyze the geometrical relationship of all clients’ local model updates. Based on our theoretical analysis, our proposed advanced free-rider attack with a carefully constructed noise can blur the difference between free-riders’ local updates and other honest updates. We also conduct experiments to demonstrate the feasibility of our attack.

**Our contributions.** We analyze the geometrical relationship of all local clients’ model updates in an iid setting, that is, the evolution trend of cosine similarity for local model update, and propose an advanced free-rider attack where the free-rider can carefully construct local model update. The forged update has the convergence property and a good stealth property.

## 2 Related work

**Defense strategies.** Existing defense methods against training-time attacks can be summarized as follows: 1) Byzantine-tolerant aggregation mechanism. It is an alternative aggregation mechanism to ensure model convergence in the presence of Byzantine participants. Traditional techniques for secure aggregation include Krum [11], GeoMed [12] and Trimmed Mean [13]. 2) Norm difference clipping. The central sever in FL system can checks the  $l_2$ -norm of each local model update returned by all clients [14]. And it clips the model updates that exceed a norm threshold. 3) Differential Privacy (DP). In recent research [15, 14], the central server can add a Gaussian noise with small standard deviations to the global model. In addition, participant-level differential privacy can reduce the effectiveness of the backdoor attack, but only at the cost of degrading the models performance on its main task. 4) Cosine Similarity. Another defense [16] targets sybil attacks by measuring the cosine similarity across the local updates and discards those that are very similar to each other. The variant version of this method is to compute the pairwise cosine similarity between all participants updates to distinguish the attackers local update. 5) Anomaly detection. The center server can detect local model updates submitted by all clients through anomaly detection methods and discards the outliers. Some cluster-based anomaly detection on multi- or high-dimensional data such as Autoencoder [17, 18], Deep Autoencoding Gaussian Mixture Model (DAGMM)[19] are often utilized.

## 3 Problem formulation

### 3.1 Federated learning

In a FL system, there is a central server which distributes its global model to all clients, and aggregates the local updates to generate a new global model at each round. We consider a typical SGD-based federated setting which consists of a central server and  $n$  clients  $C_1, C_2, \dots, C_n$ , each with  $N_i$  local private training samples. The total number of training samples is  $N$ . At each round  $t(t = 0, 1, 2, \dots)$ , the server distributes the current global model  $\theta(t)$  to all clients. Each participant  $C_i$  trains a new local model based on global model  $\theta(t)$ , and then sends back the local update  $U_i(\theta)$  to the server. Later, the server aggregates all local updates as follows:

$$\theta(t+1) = \theta(t) + \sum_{i=1}^n \frac{N_i}{N} U_i(\theta). \tag{1}$$

### 3.2 Formalizing FL training process

**The Ornstein-Uhlenbeck process definition.** In FL systems, SGD algorithm [20] is an important algorithm, which enables efficient optimization by following noisy gradients with a decreasing step size. In this work, we follow the typical FL setting equipped with SGD algorithm. According to recent researches [21, 22, 23, 24] SGD can be generally modeled as an OU process. The OU process is a stationary Gauss-Markov process where drifts towards its mean function over time. Specifically, the OU process can be described by the following stochastic differential equation:

$$d\theta_t = \lambda(\mu - \theta_t)dt + \sigma dW_t, \quad (2)$$

where  $\lambda$  denotes the magnitude of the OU process and  $W_t$  denotes the standard Wiener process. This equation means that the OU process drifts towards the mean  $\mu$  with a velocity of  $\lambda$  and a volatility driven by a Brownian motion with a variance  $\sigma$ .

**Local training process as an OU process.** Follow the prior discussion in [21, 24], we formulate the local training process with SGD algorithm as an OU process. In our setting, at round  $t$ , client  $C_i$  receives the global model  $\theta(t)$  and computes the local model update  $U_i(\theta)$ . In gradient descent, the loss function is  $\mathcal{L}(\theta; \mathcal{S}) = \frac{1}{s} \sum_{j=1}^s \ell_j(\theta)$ , where  $s$  is the size of a mini-batch dataset  $\mathcal{S} \subseteq D_i$ , and  $\ell_j(\theta)$  is the loss of a point  $s_j \in \mathcal{S}$ . The stochastic gradient is  $\nabla_{\theta} \mathcal{L}(\theta; \mathcal{S}) = \frac{1}{s} \sum_{j \in \mathcal{S}} g_{i,j}(\theta)$ , where  $g_{i,j}(\theta)$  is the stochastic gradient for a data point  $s_j$ . In local epoch  $k$  with a local model  $\theta_i(k)$  and a local learning rate  $\gamma$ , we can get

$$\theta_i(k+1) = \theta_i(k) - \frac{\gamma}{s} \sum_{j \in \mathcal{S}} g_{i,j}(\theta). \quad (3)$$

The stochastic gradient is a sum of  $s$  independent, uniformly sampled contributions and the gradient noise is Gaussian with variance  $\propto \frac{1}{s}$  [21]. According to the central limit theorem, we know that  $\nabla_{\theta} \mathcal{L}(\theta; \mathcal{S}) \rightarrow \mathcal{N}(g_i(\theta), \frac{B(\theta)B(\theta)^T}{s})$ , where  $g_i(\theta)$  is a full gradient and  $B(\theta)B(\theta)^T$  is the corresponding covariance matrix. According to previous discussion in [21], we assume that when  $\theta_i$  approaches a stationary value,  $B(\theta) = B$ , which is a constant. Thus the following equation holds:

$$\Delta\theta_i = \theta_i(k+1) - \theta_i(k) \approx -\gamma g_i(\theta) + \frac{\gamma}{\sqrt{s}} B \mathcal{N}(0, I). \quad (4)$$

This equation is a discretization equation of the continuous-time stochastic differential equation,

$$d\theta_i = -g_i(\theta)dt + \frac{\gamma}{\sqrt{s}} B dW_t. \quad (5)$$

**Assumption 1.** We assume that, for the client  $i$  with a local model  $\theta_i(t)$  that satisfies  $\theta_i(t) \xrightarrow{t \rightarrow +\infty} \theta_i$ . For convex model, the local model gradient at round  $t$  is close to

$$g_i(\theta) \approx \lambda_i(\theta_i(t) - \theta_i). \quad (6)$$

Client  $C_i$  trains a local model initiated with  $\theta(0)$  and finally the model converges to  $\theta_i$ . We assume that the local model gradient  $g_i(\theta)$  can be modeled by the OU process Eq.2, which is almost proportional to the distance between current local model  $\theta(t)$  and convergent local model  $\theta_i$ . Note that this assumption is an ideal and simple OU model for the evolution of local training process.

We can replace the parameter  $g_i(\theta)$  in Eq. 5. Then, we can have

$$d\theta_i \approx \lambda_i(\theta_i - \theta_i(t))dt + \frac{\gamma}{\sqrt{s}} B dW_t. \quad (7)$$

**Theorem 1.** The local training model's evolution can be represented by  $\theta_i(t) \approx \theta(0)e^{-\lambda_i t} + (1 - e^{-\lambda_i t})\theta_i + \frac{\gamma}{\sqrt{s}} e^{-\lambda_i t} \int_0^t e^{\lambda_i t} B dW_t$ , which satisfies  $\mathbb{E}[\theta_i(t)] \xrightarrow{t \rightarrow +\infty} \theta_i$  and  $\text{Var}[\theta_i(t)] \xrightarrow{t \rightarrow +\infty} \frac{\gamma^2 B^2}{2\lambda_i s}$ .

From the local training model's evolution  $\theta_i(t)$ , we can get that in the initial round, client  $C_i$  trained with the initial global model  $\theta(0)$  converges to the local model  $\theta_i$  with a speed of  $O(e^{-\lambda_i t})$ . The expectation of the local model weights is  $\theta_i$  and the variation for the convergent local model is  $\frac{\gamma^2 B^2}{2\lambda_i s}$ , which shows the uncertainty of the process.

**Global training process as an OU process.** As defined in section 3.1, the local model updates for  $n$  clients are  $U_1(\theta), U_2(\theta), \dots, U_i(\theta), \dots, U_n(\theta)$ . According to [25], their observation is that when the learning rate is sufficiently small, the effect of  $E$  steps of local updates is similar to one step update with a larger learning rate. Therefore, we can assume that  $U_i(\theta) \approx d\theta_i$ , that is,  $U_i(\theta) \approx d\theta_i = -g_i(\theta)dt + \frac{\gamma}{\sqrt{s}} B_i dW_t$ . For the global training process defined in Eq.1, similar to the discretization

equation and continuous-time stochastic differential equation illustrated in Eq.5, we can further get

$$d\theta(t) = \sum_{i=1}^n \frac{N_i}{N} \lambda_i (\theta_i - \theta_i(t)) dt + \sum_{i=1}^n \frac{N_i}{N} \frac{\gamma}{\sqrt{s}} B_i dW_t. \quad (8)$$

**Theorem 2.** *The global training model's evolution can be represented by  $\theta(t) \approx \theta(0)e^{-\bar{\lambda}t} + (1 - e^{-\bar{\lambda}t})\bar{\theta} + \sum_{i=1}^n \frac{N_i}{N} \frac{\gamma}{\sqrt{s}} e^{-\bar{\lambda}t} \int_0^t B_i e^{\bar{\lambda}t} dW_t$ , which satisfies  $\mathbb{E}[\theta(t)] \xrightarrow{t \rightarrow +\infty} \bar{\theta}$  and  $\text{Var}[\theta(t)] \xrightarrow{t \rightarrow +\infty} \sum_{i=1}^n \frac{\gamma^2 N_i^2 B_i^2}{2\lambda N^2 s}$ .*

We define  $\bar{\lambda} = \sum_{i=1}^n \frac{N_i \lambda_i}{N}$  and  $\bar{\theta} = \frac{\sum_{i=1}^n N_i \lambda_i \theta_i}{\sum_{i=1}^n N_i \lambda_i}$ . The global model  $\theta(t)$ , initiated with  $\theta(0)$ , finally converges to  $\bar{\theta}$  with a speed of  $O(e^{-\bar{\lambda}t})$ . For iid setting in FL, the local data can be regarded as samples drawn from the overall distribution that represents the overall distribution. Therefore, we assume that the parameter  $\lambda_i$  is the same for all clients that equals to  $\lambda$ . We can infer that  $\bar{\theta} = \sum_{i=1}^n \frac{N_i}{N} \theta_i$ , that is, the expectation of the global model  $\theta(t)$  is the weighted average of all local optimal model, and the variation for the convergent global model is  $\sum_{i=1}^n \frac{\gamma^2 N_i^2 B_i^2}{2\lambda N^2 s}$ .

## 4 Methods

### 4.1 Scaled delta free-rider attack

**Motivation.** When learning rate is small, the model update vectors in two adjacent rounds are almost the same. Thus, we use the weighted average value of all local model updates returned by honest clients  $\theta(t) - \theta(t-1)$  as forged local update. As the model tends to converge, the local update gradually decays and tends to zero. We try to simulate honest clients' behavior by using the ratio of previous two rounds'  $l_2$ -norm of local updates as the decay rate. To simulate a real global update  $\theta(t+1) - \theta(t)$  in the next round, we can forge local update in our scaled delta attack as follows:

$$U_f(\theta) = \frac{\|\theta(t) - \theta(t-1)\|}{\|\theta(t-1) - \theta(t-2)\|} (\theta(t) - \theta(t-1)). \quad (9)$$

According to the discussion in section 3.2, when the learning rate is sufficiently small, the effect of  $E$  steps of local update is similar to one step update with a larger learning rate. Thus,  $U_f(\theta)$  can be viewed as the weighted average of all local model updates with just a one-step training. In FL system,  $E = 1$  makes FedAvg equivalent to SGD[25]. Therefore, our fake update could be viewed as a global update over the whole training dataset with a one-step training.

**Convergence proof.** We analyze our scaled delta attack following the proof of convergence in [9]. We denote by  $J_1$  the set of honest clients and by  $J_2$  the set of  $m$  free-riders. The total number of training samples declared by free\_riders is  $M$ . At round  $t$ , the aggregated model with free-riders could be calculated as follows:

$$d\theta(t) \approx \sum_{i \in J_1} \frac{N_i}{N-M} \lambda_i (\theta_i - \theta_i(t)) dt + \sum_{i \in J_1} \frac{N_i}{N-M} \frac{\gamma}{\sqrt{s}} B_i dW_t. \quad (10)$$

**Theorem 3.** *The scaled delta attack has comparable performances in terms of convergence property compared to the training process involving only honest clients. We have  $\theta(t) \approx \theta(0)e^{-\bar{\lambda}t} + (1 - e^{-\bar{\lambda}t})\bar{\theta} + \sum_{i \in J_1} \frac{N_i}{(N-M)} \frac{\gamma}{\sqrt{s}} e^{-\bar{\lambda}t} \int_0^t B_i e^{\bar{\lambda}t} dW_t$ , which satisfies  $\mathbb{E}[\theta(t)] \xrightarrow{t \rightarrow +\infty} \bar{\theta}$  and  $\text{Var}[\theta(t)] \xrightarrow{t \rightarrow +\infty} \sum_{i \in J_1} \frac{\gamma^2 N_i^2 B_i^2}{2\lambda(N-M)^2 s}$ .*

We denote  $\bar{\lambda} = \sum_{i \in J_1} \frac{N_i}{N-M} \lambda_i$  and  $\bar{\theta} = \frac{\sum_{i \in J_1} N_i \lambda_i \theta_i}{\sum_{i \in J_1} N_i \lambda_i}$ . Similar to Theorem 2, we know that the convergence rate, the final global model and the uncertainty of the training process with free-riders has the same form of expression as the training process with only honest clients. Our proposed scaled delta attack has comparable convergence performance with normal FL training.

### 4.2 Advanced free-rider attack

The scaled delta attack hasn't a good stealth property and could be detected easily. In this scaled delta attack, we can obtain the weighted average update of all clients as  $\bar{U}(\theta) = \sum_{i=1}^n \frac{N_i}{N} U_i(\theta)$ , which is a high-dimensional vector. If we can find the relationship (e.g, cosine similarity and  $l_2$ -norm) between these local updates, we can construct a fake update similar to a real update.

**Analysis for cosine similarity of local updates.** In general, the expectation of local model weights for  $n$  clients  $\theta_1, \theta_2, \dots, \theta_n$  could be viewed as a  $p$ -dimensional vector,  $p$  is the dimension of the local model parameters after being flattened. As shown in Theorem 2, we know that the global model weights converge to  $\bar{\theta} = \sum_{i=1}^n \frac{N_i}{N} \theta_i$  in iid setting. And we denote  $\epsilon_i = \theta_i - \bar{\theta}$ , which demonstrates the deviation between local client model and global model, which is resulted from the difference between local datasets and whole dataset. We assume that the smallest space which contains all  $n$  vectors is  $\Omega$ , and  $\theta_i$  is a random vector which deviates from the vector  $\bar{\theta}$ . For iid setting in FL, the local data can be regarded as samples drawn from the overall distribution which represents the overall distribution. Therefore, these  $n$  vectors  $\theta_1, \theta_2, \dots, \theta_n$  could be viewed as randomly distributed points around point  $\bar{\theta}$ . For client  $C_i$ , we set  $\mathbb{E}(|\epsilon_i|) = \epsilon$ . The following theory shows the relationship between the number of training rounds  $t$  and the cosine value of the angle generated from the local model update vectors for any two clients.

**Lemma 1.** *In iid setting, the cosine value between two local updates is  $\cos \beta$ . The relationship between  $\mathbb{E}(\cos \beta)$  and the number of training rounds  $t$  satisfies the equation:  $\mathbb{E}(\cos \beta) \approx \frac{C^2}{C^2 + e^{2\lambda t}}$ .*

In Lemma 1,  $C$  is an introduced constant parameter that shows the multiple relationship between two expectations, that is,  $\mathbb{E}(|\bar{\theta} - \theta(0)|) = C\mathbb{E}(|\epsilon_i|)$ . When  $t = 0$ , the value of  $\mathbb{E}(\cos \beta) \approx 1$ , which means in the initial stage of FL training, random two local updates almost have the same direction with each other. With the training process moving forward, the value of  $\mathbb{E}(\cos \beta)$  is decreasing with a rate of  $O(e^{-\lambda t})$ . When the global model converges, that is, when  $t \rightarrow \infty$ , the value of  $\mathbb{E}(\cos \beta)$  is close to 0, that is, random two local updates are almost orthogonal to each other, which means that when the model is close to convergence, differences in local model updates caused by the specificity of local datasets will gradually appear.

**Lemma 2.** *In iid setting, the l2-norm relationship between the local model update  $U_i(\theta)$  and the weighted average value of all local model updates  $\bar{U}(\theta)$  is  $\frac{|\mathbb{E}(U_i(\theta))|}{|\mathbb{E}(\bar{U}(\theta))|} = \sqrt{\frac{n^2}{n + (n^2 - n)\mathbb{E}(\cos \beta)}}$ . And the vector difference between  $U_i(\theta)$  and  $\bar{U}(\theta)$  is orthogonal to vector  $\bar{U}(\theta)$ .*

We denote  $\bar{U}(\theta) = \theta(t) - \theta(t-1) = \sum_{i=1}^n \frac{N_i}{N} U_i(\theta)$ , which is a high-dimensional vector constructed by all local model updates. We also know how to simulate the expectation of  $\cos \beta$  for random two vectors  $U_i(\theta)$  and  $U_j(\theta)$ . Therefore, in the next section, we will show how to construct our advanced free-rider attack based on Lemma 1 and Lemma 2.

For iid setting, according to previous discussion in section 4.1, we know that our fake update  $U_f(\theta) = \frac{\|\theta(t) - \theta(t-1)\|}{\|\theta(t-1) - \theta(t-2)\|} (\theta(t) - \theta(t-1))$  could be viewed as a global update over the whole training dataset with one-step training, and approximately equal to the weighted average value of all local model updates in the next round. Because of the difference between local datasets and whole

dataset, there exists weight divergence between local client model and global model. Then, we analyze and quantify the weight divergence in Lemma 2. Thus, by exploiting the conclusion that in high-dimensional spaces, random vectors are orthogonal[26], our advanced strategy is to add Gaussian noise  $\varphi(t)$  satisfying  $|\varphi(t)| = \sqrt{\frac{n^2}{n + (n^2 - n)\mathbb{E}(\cos \beta)} - 1} |U_f(\theta)|$ , which is a simulated deviation between  $U_f(\theta)$  and a real local model update  $U_i(\theta)$ . The advanced free-rider model update is constructed by equation  $\hat{U}_f(\theta) = U_f(\theta) + \varphi(t)$ . The details of advanced free-rider attack are shown in Algorithm 1. The attacker firstly calculates the expectations of global updates to estimate parameters  $\bar{\lambda}$  and  $\cos \beta$ . Then it chooses a random fraction of dimensions in the scaled delta update to add Gaussian noise, which simulate specificity of the training on local dataset.

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**Algorithm 1:** AdvancedAttack. The malicious client  $C_f$  carries out the advanced free-rider attack based on global model  $\theta(t)$  received at round  $t$  and  $\theta(t-1)$  at round  $t-1$ .

- 1: **Parameter estimation.**
- 2:  $l(t) \leftarrow |\mathbb{E}(\theta(t) - \theta(t-1))|$
- 3:  $l(1) \leftarrow |\mathbb{E}(\theta(1) - \theta(0))|$
- 4:  $\bar{\lambda} \leftarrow \ln \sqrt[t-1]{\frac{l(t)}{l(1)}}$
- 5:  $\mathbb{E}(\cos \beta) \approx \frac{C^2}{C^2 + e^{2\lambda t}}$
- 6: **Update generation.**
- 7:  $U_f(\theta) = \frac{\|\theta(t) - \theta(t-1)\|}{\|\theta(t-1) - \theta(t-2)\|} (\theta(t) - \theta(t-1))$
- 8:  $|\varphi(t)| = \sqrt{\frac{n^2}{n + (n^2 - n)\mathbb{E}(\cos \beta)} - 1} |\mathbb{E}(U_f(\theta))|$
- 9: Choose a fraction of parameter dimension as  $d$
- 10: Add Gaussian noise  $|\varphi(t)|\mathcal{N}(0, \frac{1}{d})$  to  $U_f(\theta)$
- 11: **Return** the constructed model update  $\hat{U}_f(\theta)$  to server

For highly skewed non-iid data, the local updates are naturally non-iid with each other. We cannot find certain geometric relationship between all local updates. Our Lemma 1 and Lemma 2 cannot be applied. However, due to the heterogeneity of local data, the local updates are spreaded in a large space which is beneficial for a free-rider to hide its fake update. We still use the advanced attack to carry out the experiments for non-iid setting.

**Convergence proof.** Similar to the proof of Eq. 10, we could have

$$d\theta(t) \approx \sum_{i \in J_1} \frac{N_i}{N-M} \lambda_i (\theta_i - \theta_i(t)) dt + \sum_{i \in J_1} \frac{N_i}{N-M} \frac{\gamma}{\sqrt{s}} B_i dW_{t_1} + \frac{M}{N-M} \varphi(t) dW_{t_2}. \quad (11)$$

**Theorem 4.** *Our proposed advanced attack has comparable performance in terms of convergence property compared to the training process involving only honest clients. We have  $\theta(t) \approx \theta(0)e^{-\bar{\lambda}t} + (1 - e^{-\bar{\lambda}t})\bar{\theta} + \sum_{i \in J_1} \frac{N_i}{N-M} \frac{\gamma}{\sqrt{s}} e^{-\bar{\lambda}t} \int_0^t B_i e^{\bar{\lambda}t} dW_{t_1} + \frac{M}{N-M} e^{-\bar{\lambda}t} \int_0^t \varphi(t) e^{\lambda t} dW_{t_2}$ , which satisfies  $\mathbb{E}[\theta(t)] \xrightarrow{t \rightarrow +\infty} \bar{\theta}$  and  $Var[\theta(t)] \xrightarrow{t \rightarrow +\infty} \sum_{i \in J_1} \frac{\gamma^2 N_i^2 B_i^2}{2\bar{\lambda}(N-M)^2 s}$ .*

We denote  $\bar{\lambda} = \sum_{i \in J_1} \frac{N_i}{N-M} \lambda_i$  and  $\bar{\theta} = \frac{\sum_{i \in J_1} N_i \lambda_i \theta_i}{\sum_{i \in J_1} N_i \lambda_i}$ . Similar to Theorem 3, we get the same conclusion that our proposed advanced attack also have comparable convergence performance with normal FL training.

## 5 Performance evaluation

The goal of our experiment is to highlight that, compared with the state of the art free-rider attacks, our advanced free-rider attack has a good performance in terms of convergence and stealth property. We conduct experiments on real world datasets and simulate a FL environment. The resulting model under advanced attack has comparable performances and convergence rate with respect to the one of the model obtained with honest clients only. Besides, the results demonstrate that in both iid and non-iid settings, our advanced attack can stay stealthy under different indicative features. Furthermore, we utilize DAGMM as an anomaly detection method to defense and evaluate our advanced attack.

**Tasks.** We evaluate our advanced free-rider attack on two image classification tasks: **1) Task1.** MNIST[27] with multi-layer perceptron(MLP)[28], **2) Task2.** Fashion-MNIST[29] with convolutional neural network(CNN). For each task, we maintain the same experimental setting composed by 50 honest clients, and we set the number of free-riders to respectively 1, 50 and 150 in both iid and non-iid setting. Each client has the same number of training samples. Details of dataset and experimental setups are provided in the appendix.

**Compared attacks.** We compare latest free-rider attacks as follows: **1) Plain free-rider.** It is the simplest way to construct fake local update. The free-rider directly returns the received global model without any local training. Since this attack is trivial for detection, we use it as a baseline strategy for free-riders. **2) Disguised free-rider.** This attack is based on additive stochastic perturbations. Specifically, the free-rider utilizes GMM to fit a multimodal distribution forms for the local update. **3) Delta weights.** This attack subtracts the current global model to the previous global model.

**Features.** Byzantine-tolerant aggregation mechanisms and DP-based defense methods focus on mitigating the influence of the attacker, they don't detect the attacker and aren't suitable for free-rider attacks. Thus, we only consider following three indicative features which can distinguish free-riders: **1) Cosine similarity.** We compute cosine values between other clients' local model updates and a randomly chosen client's update vector. **2) L2-norm.** We compute  $l_2$ -norm of every clients' local model updates. **3) Standard eviation(STD).** We compute the std value of each clients' local model vector.

**DAGMM.** This is a model for unsupervised anomaly detection. The model contains two main components: a compression network and an estimation network. the compression network performs dimensionality reduction for input samples by a deep autoencoder, prepares their low-dimensional representations from both the reduced space and the reconstruction error features, and feeds the representations to the subsequent estimation network; Then, the estimate the network obtains the feeds and predicts their energy in the Gaussian Mixture Model (GMM) framework.

**Results.** The experimental results for both non-iid setting of **Task1** and **Task2** can be found in the

**Convergence and performances.**

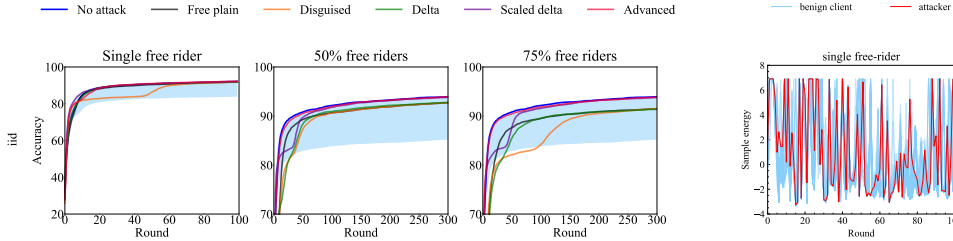


Figure 1: Accuracy performance in iid setting of MNIST dataset Figure 2: Sample energy in iid setting of MNIST dataset

As shown in Figure 1, we show the evolution of the global model’s accuracy for iid setting in three scenarios where the number of free-riders is 1 and accounts for 50% and 75% of all clients. The shaded blue region indicates the variability of FL model with only honest clients which is estimated from 10 different training initialization <sup>2</sup>.

The results indicate that, independently from the chosen free-rider attack, the resulting global models have comparable performances compared with the global model obtained with only honest clients. However, the convergence speed of different attacks is different. In 50% free-rider scenarios and 75% free-rider scenarios, the convergence rate of other attacks such as plain free-rider, disguised free-rider and delta weights is significantly lower than the normal convergence rate. However, our scaled delta and advanced free-rider attacks have comparable performances in terms of convergence rate compared to the training process involving only honest clients. This result is also in agreement with Theorem 3 and Theorem 4 which suggest that the accuracy and convergence rate of the final global model is not affected by the number of free-riders.

**Stealth property.** In this section, we study the stealth property of various attack methods by comparing indicative features. Since plain free-rider attack is easy to be detected, we only compare our attack methods with disguised free-rider and delta weights attacks.

As shown in Figure 3, we observe that the local update of our advanced attack achieves stealth property in all indicators. The other attack methods do not have a good stealth property. Specifically, the evolution of cosine similarity and  $l_2$ -norm of our advanced attack confirms Lemma 1 and Lemma 2. Note that in the initial several rounds, our advanced attack participate the training process honestly to compute core parameters such as  $C$  which can be used in the simulation of local update.

**Anomaly detection.** To defend our advanced attack, we also utilize a popular anomaly detection model DAGMM. The results of all sample energy values are shown in Figure 2. The sample energy values of free-riders are hidden in the energy value area of other honest clients in iid setting, which is difficult to distinguish by DAGMM. The results indicate that our advanced attack is powerful and can escape from the cluster-based anomaly detection techniques.

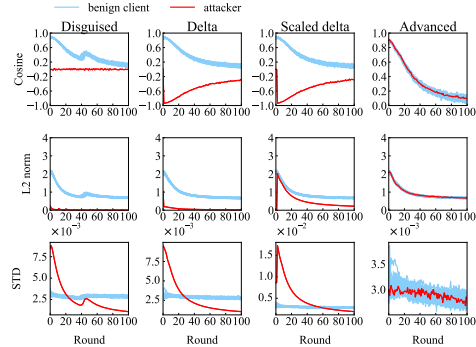


Figure 3: Features in iid setting of MNIST dataset

## 6 Conclusion and future work

In this paper, we propose scaled delta attack which can guarantee a convergent global model. Then we also present our advanced free-rider attack by improving the stealth property of scaled delta attack. Detailed experiments validate that our advanced free-rider attack has good performance and maintain stealth against existing defense strategies. For future work, we would like to study how to carry out our free-rider attacks in a real-world setting(such as partial client participation, secure aggregation[30], etc.), and investigate more defense strategies to protect FL systems against advanced free-rider attack.

<sup>2</sup>In our experiment for MNIST, the maximum global epoch for the training process is 300 due to our limited computation resources

## Acknowledgements

This work is financially supported by National Natural Science foundation of China under Grant No. 61732022 and National Key R&D Program of China under Grant No. 2017YFB0803002.

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